Mine Planning Under Uncertainty Constraints
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INTRODUCTION

The fundamental input to pit optimization and mine planning is the resource estimate. Reserve reports, pit designs, project debt financing, and operating plans are all contingent upon the information derived from the pit optimization process. Failure to recognize the risk associated with the resource estimate can lead to lost investment capital or lost opportunity (Figure 1).

In 1999, a mining company faces many obstacles to the basic goal of mining and recovering a commodity at a profit. A strong US economy and low commodity prices has diverted investment dollars from the mining industry to higher growth and earnings opportunities in the stock market. Furthermore, low commodity prices have reduced the amount of risk that lending banks are willing to accept. In the North American gold mining industry, after more than a decade of relatively low cost production from heap leaching near surface oxide deposits, the deposits being taken into production today require more capital intensive processing facilities, face higher operating costs, and consequently lower profit margins. To prosper into the next millennium, today's mining company must make better decisions regarding the reporting of reserves and mine operations than in the past. Project risk must be quantified both for internal planning and external financing purposes, and the reconciliation process becomes more important with declining profit margins.

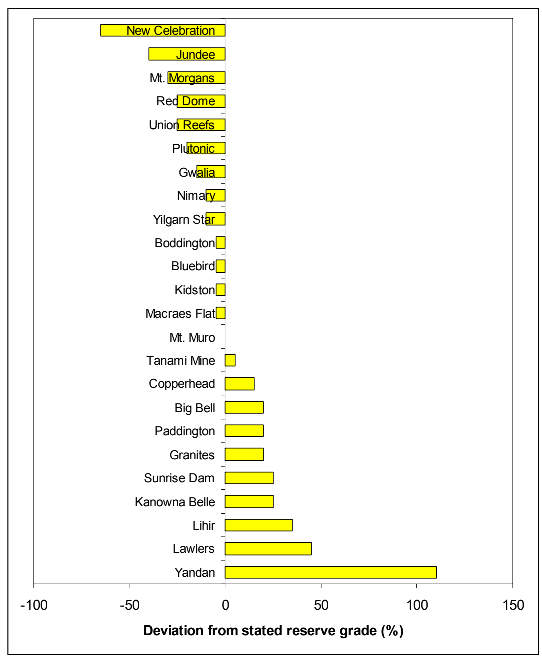


Figure 1 Reserve grade variation to production grade (From Baker and Giacomo, 1998).

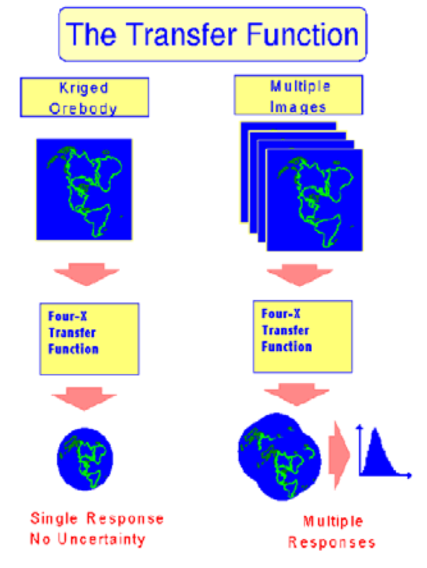
This paper focuses on how the inherent deposit variation in grade, modelled using conditional simulation, can be used to assess confidence in reserves, time to capital payback and potential mine life cash flow variation, pit design options, and operating plan accuracy.

METHODOLOGY FOR MODELLING RISK

Geostatistical conditional simulations are becoming standard industry tools for the evaluation of uncertainty and therefore risk. This is accomplished by building a model of the deposit that reproduces faithfully the full histogram and variogram of the conditioning data. Therefore, these models honor the spatial variability of the deposit as represented by the existing data. Some of the more important aspects of building the conditional simulation models have been published elsewhere, including among others Rossi (1999), Rossi and Van Brunt (1997), and Goovaerts (1996).

Multiple simulations (models of the deposit) are possible because of the random nature of the stochastic (Monte-Carlo) process involved in developing each simulation. In the case study developed below, 11 simulations were obtained. These 11 simulations represent the possible range of grade values for each block. The set of possible grades for each block is in fact a probability function curve, and is all that is needed (under the model) to evaluate risk. Not only E-type estimates can be obtained (as an average of the simulations, in theory equating to kriging), but also probabilities of exceeding or not exceeding thresholds. Typically it is more useful to use probability intervals as measures of risk. In layman terms, these probabilities are sometimes expressed as “confidence intervals”; in strict sense, they describe the range of values (minimum and maximum) that under the model chosen represent the upper and lower boundaries of probability intervals. For example, one would say, for an individual block, that there is a 80% probability that the block true, unknown values is between 0.6 and 1.1g/t. This particular block would be more “certain” compared to another where the same probability interval may have a 0.2 and 2.0g/t lower and upper bounds. This analysis can be generalized to specific areas within the deposit, or the deposit as a whole. If each simulation is passed through a pit optimizer such as Whittle Four-X, then a set of 11 alternative pits will be obtained. This is the Transfer Function described in Rossi and Van Brunt (1997), and it allows us to measure risk at the mine planning and mine evaluation stage properly accounting for geologic and grade uncertainty.

Figure 2 Transfer function and risk modelling



THE SIMULATION MODELS

The data set used consisted of gold and copper grades, plus a lithology code. The lithologies involved that contained some mineralization were an intrusive unit (INT, low grade), and a high-grade core, simply referred to here as HI.

A total of approximately 11,000 assays yielded about 2,750 20ft composites. Both gold and copper show strong bimodality, according to the lithologic boundaries. The distributions of both gold and copper are skewed, with the 20ft copper composites showing the highest coefficient of variation, at 1.9.

The data set was first declustered using the cell declustering method (Deutsch and Journel, 1992), and then transformed to a standard Gaussian distribution. This was done because the simulation method requires that the transformed Gaussian distribution of the composites be used.

Semi variograms of the transformed and untransformed composites were obtained for both gold and copper. These variograms were modelled, and then used as input parameters into the simulation algorithm. This was done independently for each rock type modelled (INT and HI), so there were four variogram models in

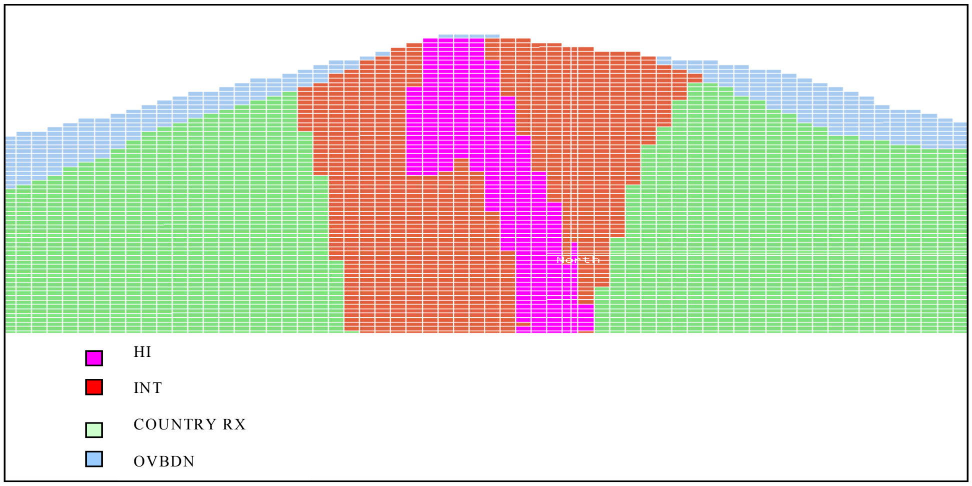
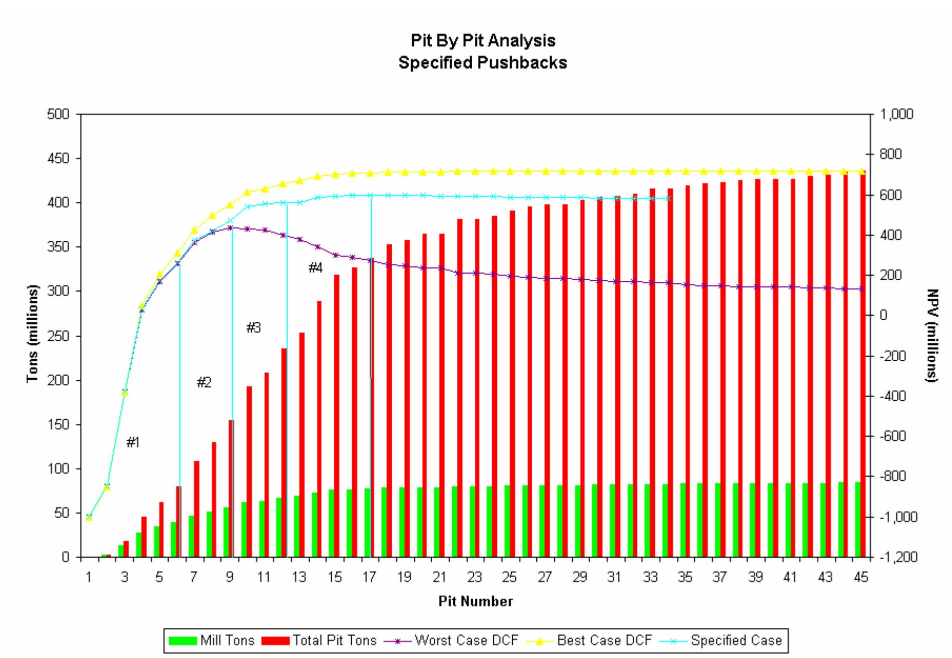


Figure 3 Generalized geologic cross section

total. For the transformed variable, ranges vary from 200 to almost 500 feet between copper and gold; the INT unit shows a slightly longer range than the HI unit. In all cases, relative nugget effects are quite reasonable, in the order of 15 to 25% of the total variance.

The conditional simulations were prepared simulating separately the INT and HI units. The simulation algorithm used was the Sequential Gaussian (Isaaks, 1990). The simulations were run on a regular grid and then cut by a solid model representing the limits of the unit, first for the INT unit (more massive) and then for the HI (core) unit. The total number of nodes simulated per rock type exceeded 3 million, although this number was significantly reduced after discarding simulated points outside the three-dimensional solids representing the lithologic units. A twelfth model was constructed by averaging the gold and copper grades from each of the 11 conditionally simulated models, this model is referred to as the E-type model.

Figure 4 E-type optimization pit shells – NPV analysis



E-TYPE OPTIMIZATION

The E-type estimate was used to develop nested pit shells using FXOP. These shells were in turn analyzed to develop a sequence of pushbacks with FXAN (Figures 4 and 5). The pushbacks were chosen in a fashion to produce approximately equal tonnage phases. Although tonnage remains approximately constant for each of the four pushbacks, the strip ratio increases and more scheduling would be required to achieve a true optimal design. The economic and operating parameters used in the analysis are presented in Table 1. These pushbacks have been modified with FXMI to honor a 150' minimum mining width. The tons and grade of ore and waste contained in the mining width adjusted shell are listed in Table 2.

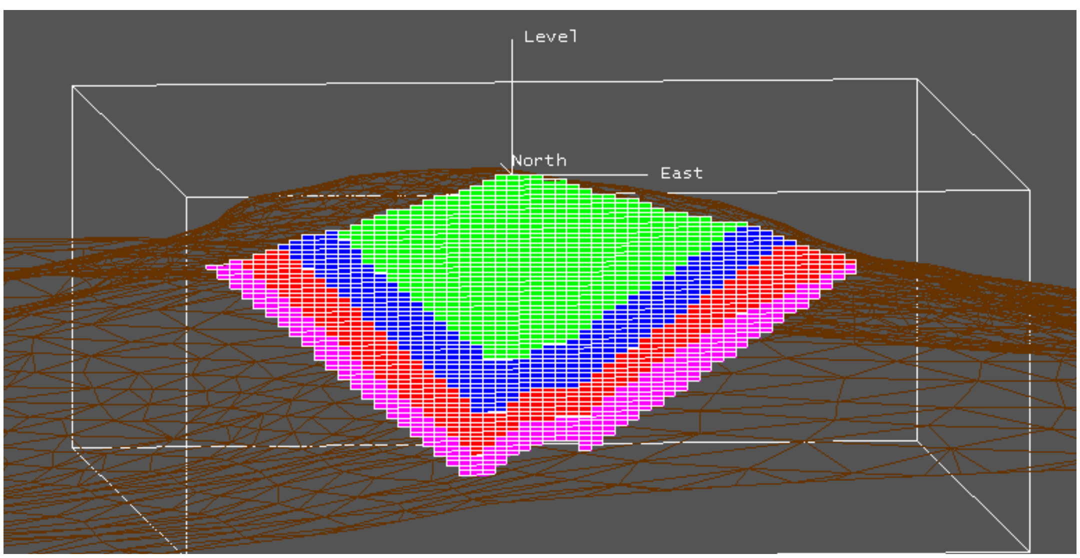
Table 1 Optimization and analysis parameters

Parameter	Value
Initial Capital	**1000M**
Time Costs	**8.4M**
Mining Cost	**1.00**
Time Cost in Processing Cost	**2.10**
Copper Price ($/10 kg)	**12.00**
Gold Price ($/troy ounce)	**250.00**
DCR	**12%**
Annual Mining Limit (tons)	**24.5M**
Annual Processing Limit (tons)	**7.0M**

Table 2 E-type optimum pit summary

Rock	Element	Tons	Metal	Grade
		(000's)	(000's)	
Waste		**237,324**		
HI		**23,781**		
	CU		**3,158**	**0.133**
	AU		**13,986**	**0.588**
INT		**35,581**		
	CU		**1,662**	**0.047**
	AU		**3,012**	**0.085**

Figure 5 Perspective view of pushbacks



RESERVE REPORTING

Amos (1998) identifies the quantity of reserves and resources as a major factor governing the level of debt funding available for a project. Amos points out that the applicant will be disadvantaged if unable to identify and categorize the risk associated with the reported tonnage and grade. One objective in using conditionally simulated models as input into Four-X, is to gain a better understanding of the risk or potential upside in a reported resource or reserve. Most if not all reserve reporting guidelines have levels of resources or reserves based on the increasing level of geological data, knowledge of the ore body, and confidence in the estimate. However none of the American, Australian, or Canadian reporting schemes which are recognized by the government securities commissions provide insight to a quantitative measure of confidence to make the distinction between levels of resources or reserves. Table 3 presents information about the contained resource in the mining width adjusted pit shell developed from the E-type estimate. Classification into specific levels of a resource should be made at the block level, prior to optimization. Most mining companies classify their Reserves and Resources based on drill hole separation and the number of drill holes within the search neighborhood, which is used in the interpolation of the block grade. Although increased drill hole density generally corresponds to increased confidence it can be easily demonstrated that the grade of the drill hole composites in the search neighborhood also contributes to the level of confidence in the block grade estimate. To truly define the confidence in a block grade and the level of resource classification this possible variation in grade must me accounted for. Table 3 presents the difference relative to the E-type estimate in the contained tons and grade of ore within the mining width adjusted pit shell using simulated copper and gold values. The cut-off grade used in each tabulation is that based on a Revenue Factor equal to one. The values in the table may be considered F_1 factors (Rossi, 1999), representing the anticipated ratio of planned to mined values. In an open pit mine the minimum increment size for expanding a reserve is a minable phase. It is therefore appropriate to look at the resource variation on a phase by phase basis (Figures 6 – 9). At this scale the incremental cash flow becomes the deciding factor as to whether the phase is included in the reserve or not.

For the purpose of classifying the resource into Measured-Indicated-Inferred categories grade variation on the block level should be evaluated. For the purpose of evaluating the risk involved in mining a given phase or pushback it is instructive to examine the Max%Diff and Excel's PERCENTRANK values in Table 4. The Max%Diff value measures the spread, or in some sense, the variance of the simulated values relative to the E-type value. The PERCENTRANK value identifies the position of the E-type value within the distribution of conditionally simulated values, and therefore measures the likelihood of realizing this value during mining.

Table 3 F_1 total resource factors

Simulations	Ore Tons %	Cu Grade %	Cu Metal %	Au Grade %	Au Metal %	Strip Ratio
Max	104.57	120.55	120.84	124.07	129.74	103.59
Min	97.23	75.66	78.95	89.05	89.02	99.64
Average	100.54	101.50	101.99	100.05	100.66	95.47
E-type	100.00	100.00	100.00	100.00	100.00	100.00
Range %	+4.57	+20.55	+20.84	+24.07	+29.74	+3.59
	-2.77	-24.34	-21.05	-10.95	-10.98	-0.36

The following conclusions are drawn from examining the resource with simulated models:

- The average of the results based on conditionally simulated grades is approximately equal to the E-type results, thereby validating the procedures followed. In a study where the E-type or kriged model is developed independently from the simulations this approach can be used to prove that the simulated results are unbiased.
- The differing variability of elements within the two rock types is easily understood.
- It follows then, that drill spacing adequate to define one element to a certain confidence and corresponding classification, may not be adequate for a second element.
- The distribution of possible results is not always distributed evenly about the E-type estimate (Figures 6 – 9).
- The PERCENTRANK function gives an indication as to the likelihood of the E-type estimate being realized (Table 4).
- With the exception of Phase 4, there is a greater likelihood of realizing or exceeding the results predicted by the E-type estimate for gold than for copper.
- Again with the exception of Phase 4, confidence is higher for the INT rock type than for the HI rock type for copper and for gold.
- The E-type results for gold in rock type HI are generally more risk free than for copper. The combination of the PERCENTRANK value for copper and the wide range of possible values leads to a lower level of confidence, and should lead to a review of the estimation parameters for copper. If the estimation parameters are valid, then additional drilling information should be collected to help reduce the risk.

This section of the paper was completed by first exporting Model Files for each pair of simulated variables from the general mine planning package. Using FXRE these Model Files were combined with a Pit List File generated from the mining width adjusted E-type Results File, into a new Results File. FXUT was then used to generate grade - tonnage information by phase, which was passed on to EXCEL for graphing.

FINANCING, COMPLETION TESTS AND OPERATING PLANS

No large scale mining project financing has been funded to date without a completion guarantee from the project sponsor. A typical completion test might include a production test that ensures:

1) Before completion, a minimum tonnage to be processed is delivered to a stockpile or leach pad.
2) A minimum average daily mined tonnage is achieved.
3) A minimum average daily ore tonnage is processed.
4) Ore grades brought to processing are within mine plan designated variation.
5) Minimum quantity and quality of product during completion.

The variability of the mineralization greatly affects each of these aspects of the completion test. Since the foundation of the supporting feasibility study is the reserve and the proposed mine plan, how accurately the mine plan predicts the actual production profile is the most critical factor in satisfying the completion test.

In Rossi and Van Brunt (1997) several methods were proposed to examine how conditionally simulated models can be used to evaluate the likelihood of realizing the results anticipated from the E-type or kriging based operating mine plan in the area of head grade and processing rates. The simulated models can also be used to evaluate cash flow risk. It is important to recognize that the project cash flow is a non-linear function of the varying quantity of resource contained in the simulated models.

In this paper the risk to project cash flow, payback, and other important project parameters are examined by passing conditionally simulated models through the E-type mine schedule and then reviewing the variability of the specific parameter.

Figure 6 Gold resource variation in HI rock type

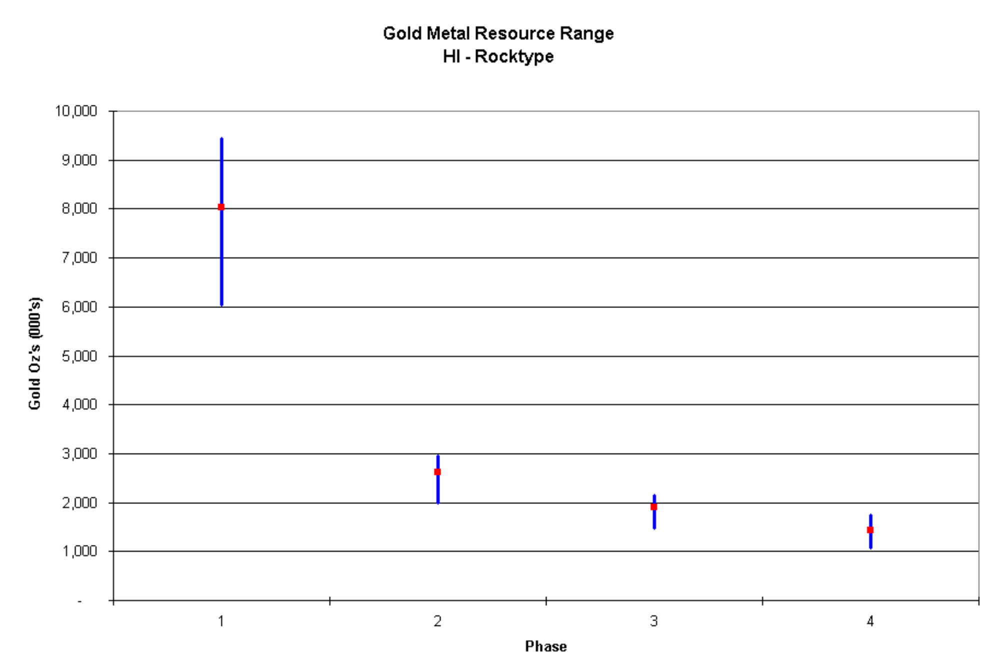


Figure 7 Copper resource variation in HI rock type

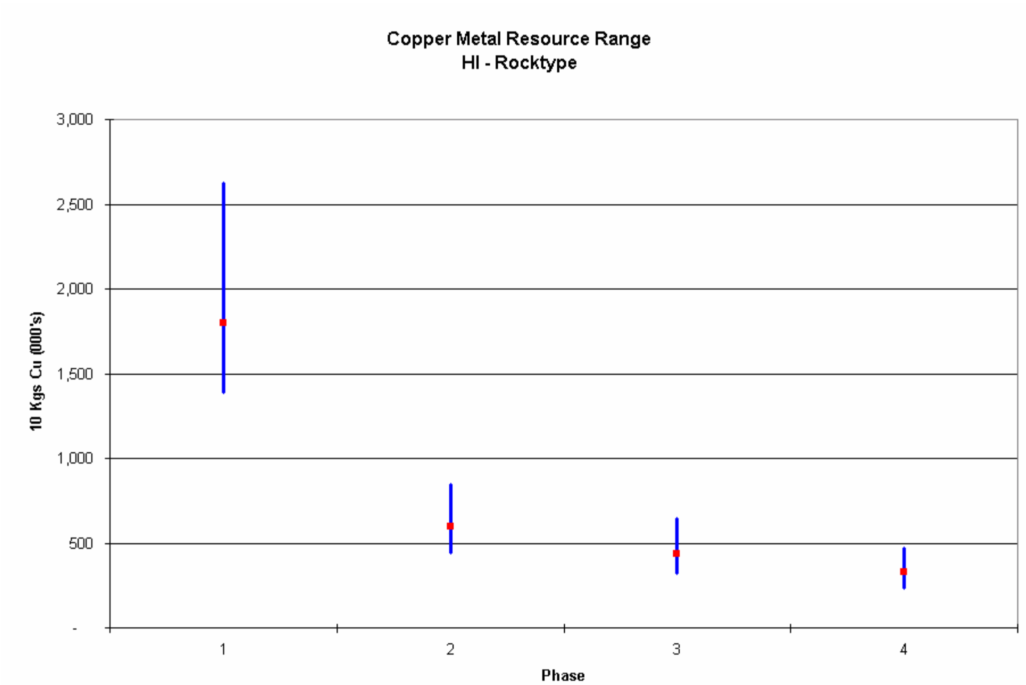


Figure 8 Gold resource variation in INT rock type

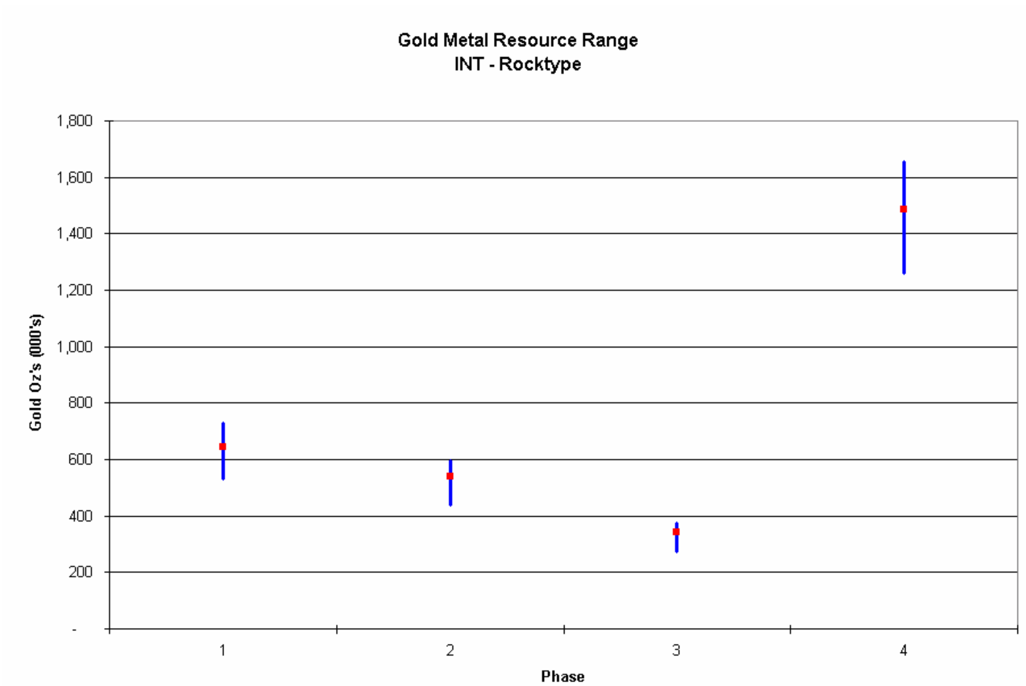


Figure 9 Copper resource variation in INT rock type

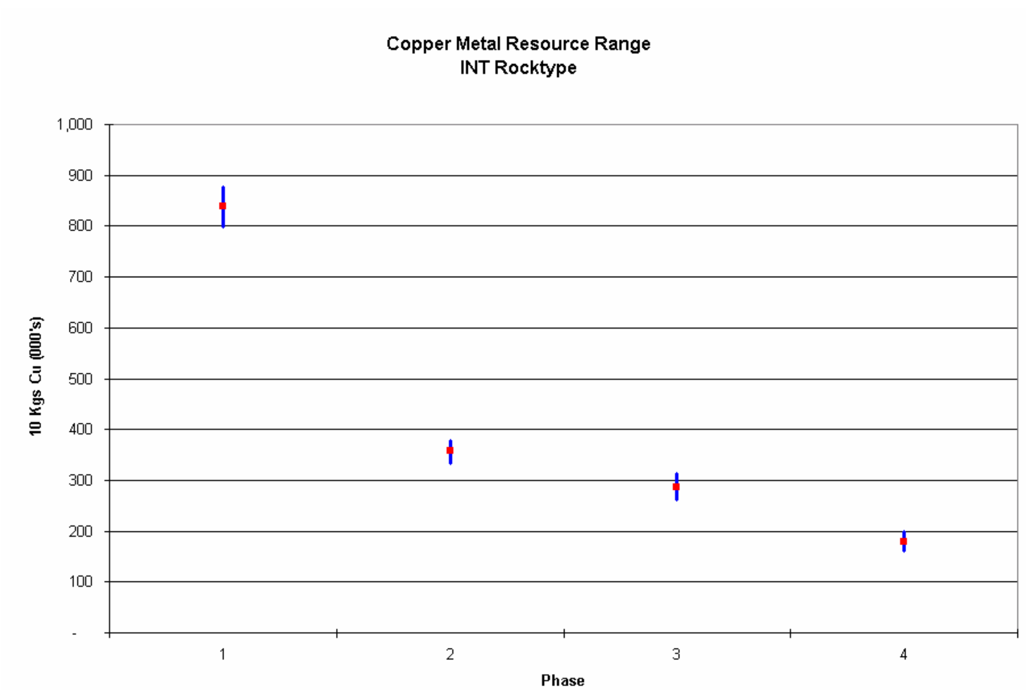


Table 4 Resource information listed by phase

Phase Rock		Total Tonnes x1000	CU Element x1000	CU Average	AU Element x1000	AU Average
PHASE_1						
Waste	Max	46,079				
	Min	44,416				
	E_Type	45,036				
HI	Max		2,625	0.194	9,438	0.699
	Min		1,389	0.103	6,040	0.447
	E_Type		1,797	0.133	8,026	0.594
	PERCENTRANK		0.703	0.700	0.522	0.523
	Max % Diff		46%	46%	25%	25%
INT	Max		877	0.051	1,656	0.089
	Min		799	0.045	1,259	0.074
	E_Type		840	0.046	1,486	0.082
	PERCENTRANK		0.557	0.166	0.381	0.250
	Max % Diff		5%	11%	15%	10%
PHASE_2						
Waste	Max	56,367				
	Min	55,436				
	E_Type	55,792				
HI	Max		846	0.185	2,959	0.648
	Min		443	0.097	2,000	0.439
	E_Type		597	0.131	2,623	0.574
	PERCENTRANK		0.709	0.708	0.441	0.440
	Max % Diff		42%	41%	24%	24%
INT	Max		377	0.052	728	0.090
	Min		334	0.045	532	0.075
	E_Type		357	0.046	645	0.084
	PERCENTRANK		0.472	0.500	0.616	0.450
	Max % Diff		6%	13%	18%	11%
PHASE_3						
Waste	Max	77,946				
	Min	76,909				
	E_Type	77,333				
HI	Max		643	0.197	2,154	0.659
	Min		320	0.098	1,476	0.453
	E_Type		435	0.133	1,904	0.583
	PERCENTRANK		0.650	0.650	0.421	0.426
	Max % Diff		48%	48%	22%	22%
INT	Max		313	0.053	597	0.094
	Min		261	0.045	437	0.079
	E_Type		287	0.047	538	0.089
	PERCENTRANK		0.480	0.500	0.583	0.350
	Max % Diff		9%	13%	19%	11%
PHASE_4						
Waste	Max	59,631				
	Min	58,871				
	E_Type	59,163				
HI	Max		470	0.193	1,744	0.715
	Min		233	0.095	1,085	0.445
	E_Type		328	0.135	1,433	0.587
	PERCENTRANK		0.500	0.500	0.670	0.669
	Max % Diff		43%	43%	24%	24%
INT	Max		198	0.053	375	0.101
	Min		161	0.045	273	0.082
	E_Type		178	0.048	344	0.092
	PERCENTRANK		0.540	0.600	0.450	0.466
	Max % Diff		11%	10%	21%	11%

Table 5 E-type model operating plan results

	Undiscounted Cash Flow	Discounted Cash Flow
Initial Capital	-1,000,000	-1,000,000
Selling Costs	-162,909	-93,169
Rehabilitation Costs	-65,985	-39,068
Timecost	-105,402	-53,143
Total	1,156,103	263,546
IRR%	18.81	
Mine Life (yrs)	12.55	
s/r	4.00	

Figure 10 E-type model annual operating plan

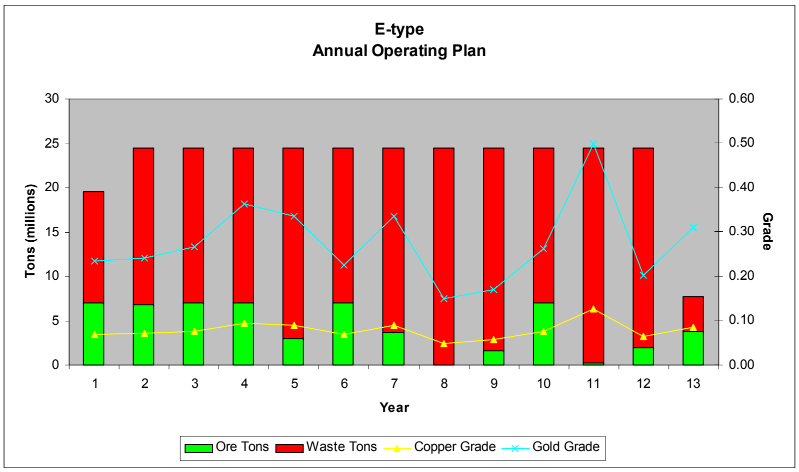


Figure 11 Operating plan variance

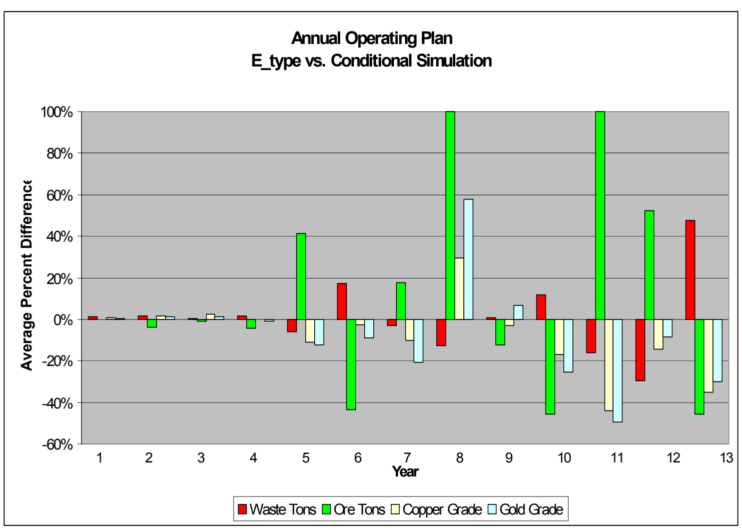
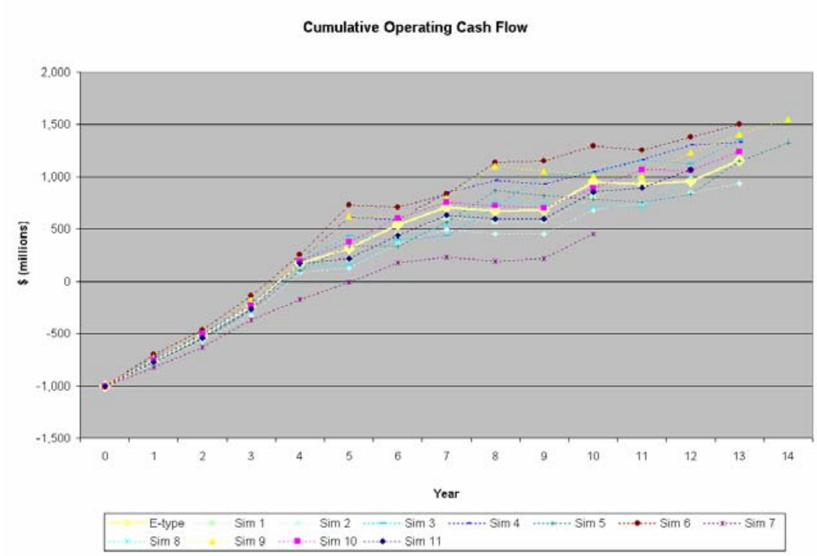


Table 5 shows the results generated from analyzing the E-type model based mine plan. Figure 10 presents the production profile for the same design. Key points in this schedule are the initial payback period (years 1 - 4) and the beginning of the later phases (periods 5, 8, and 11). Figure 11 shows that the production profile in years 1 through 4 is very stable. The payback schedule from the E-type estimate is low-risk.
Later in the schedule, specifically years 8 and 11 the schedule suffers from a lack of ore tons. This production short fall results in a very high variance when the simulated models introduce higher amounts of tons. The actual variance being measured here consists therefore of a component attributable to the variability in the mineralization, as well as a component attributable to the quality of the schedule at this stage.

Another important aspect in securing financing is the ability of the mining company to demonstrate a certain amount of debt service coverage overall, and in downside scenarios. The cash flow coverages in downside scenarios drive acceptable capital structures and debt capacities in the financing. Downside multiples may vary between 1.1 - 2.0 of the debt. Base multiples may vary 1.3 to 4.0, dependent upon bank policy, prevailing economic conditions, and other risk.

Figure 12 shows the cumulative undiscounted cash flow over the mine life for the E-type model based schedule and for all of the individual simulations. The graph gives a good indication of the potential range in project cash flow by the magnitude of the vertical spread of the individual lines at the end of the E-type planned mine life in year 13. The graph also gives a good indication of the likelihood of payback occurring when anticipated from the E-type schedule by the magnitude of the horizontal spread of the cash flows when crossing the zero cumulative cash flow gridline.

Figure 12 Cash flow versus time



PIT DESIGN

Producing a detailed design from an optimal outline involves smoothing, while deviating as little as possible from the optimal outline. As was indicated by the pit-by-pit graph in Figure 4, minor deviations from the optimal outline will have little impact on the total cash flow as long as any additional waste mined is offset by the ore which it covers.

It is common practice to assess the economic stability of pit walls by varying the commodity price, where significant movements in the position of the wall indicate that the material in question is marginal. Equally important, but perhaps more difficult to ascertain, is to identify the portions of the pit which are "carried" by relatively erratic mineralization or mineralization that is not as well defined as most of the pit. There is no procedure for an E-type or kriged estimate that can address this problem.

The eleven pairs of simulated copper and gold values were passed on to FXOP for optimization. The Specified Case pushback selection from the E-type estimate was then imposed upon the simulated Results File, and FXMI was run to adjust the shells for minimum mining width. The mining width adjusted Results Files can then be loaded into the general mine planning package, and the conditional probability of each block being mined according to the E-Type mine plan may be calculated. Conditional probability of mining maps may be developed by bench or section and used as guides during pit design. Figures 13 through 15 show bench plans of the probability of each block being mined according to the E-type model based mine plan. The maps suggest that the following points should be incorporated into the design:

- Reduce the size of the initial phases by

Figure 13 5810 bench mining probability map

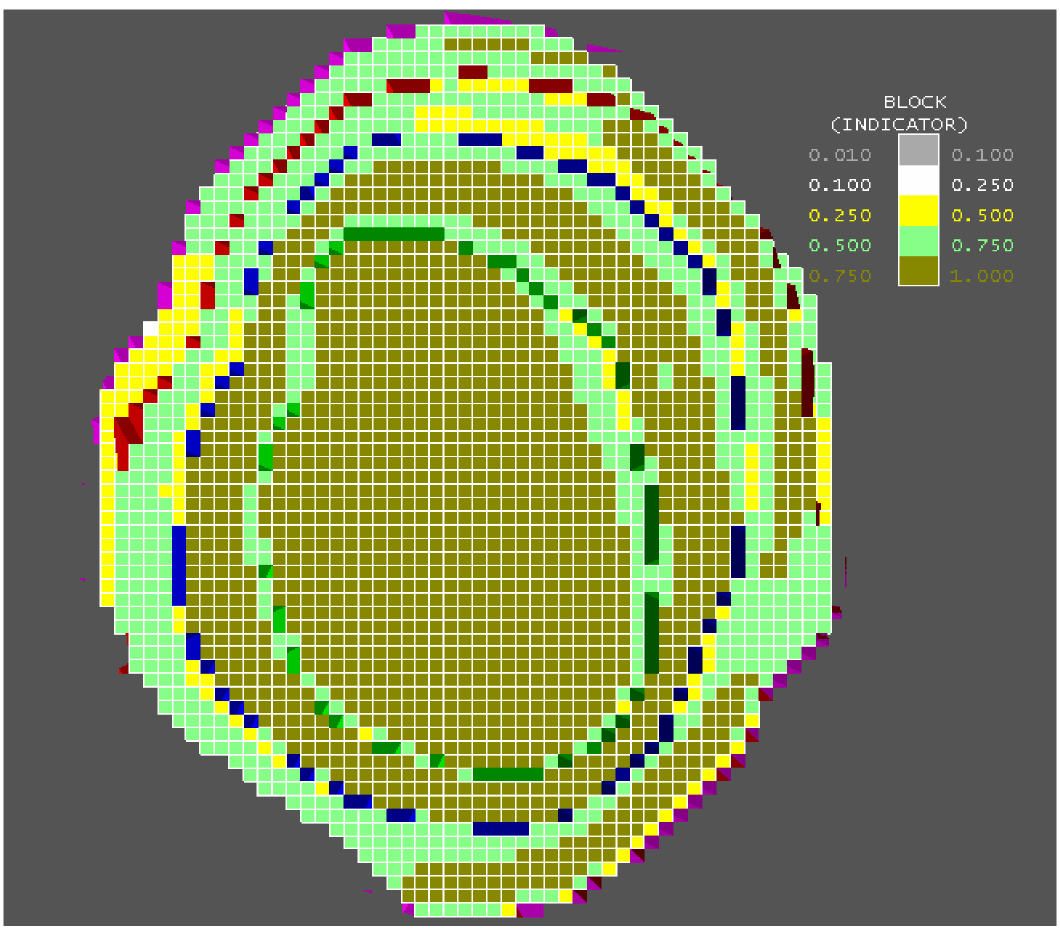
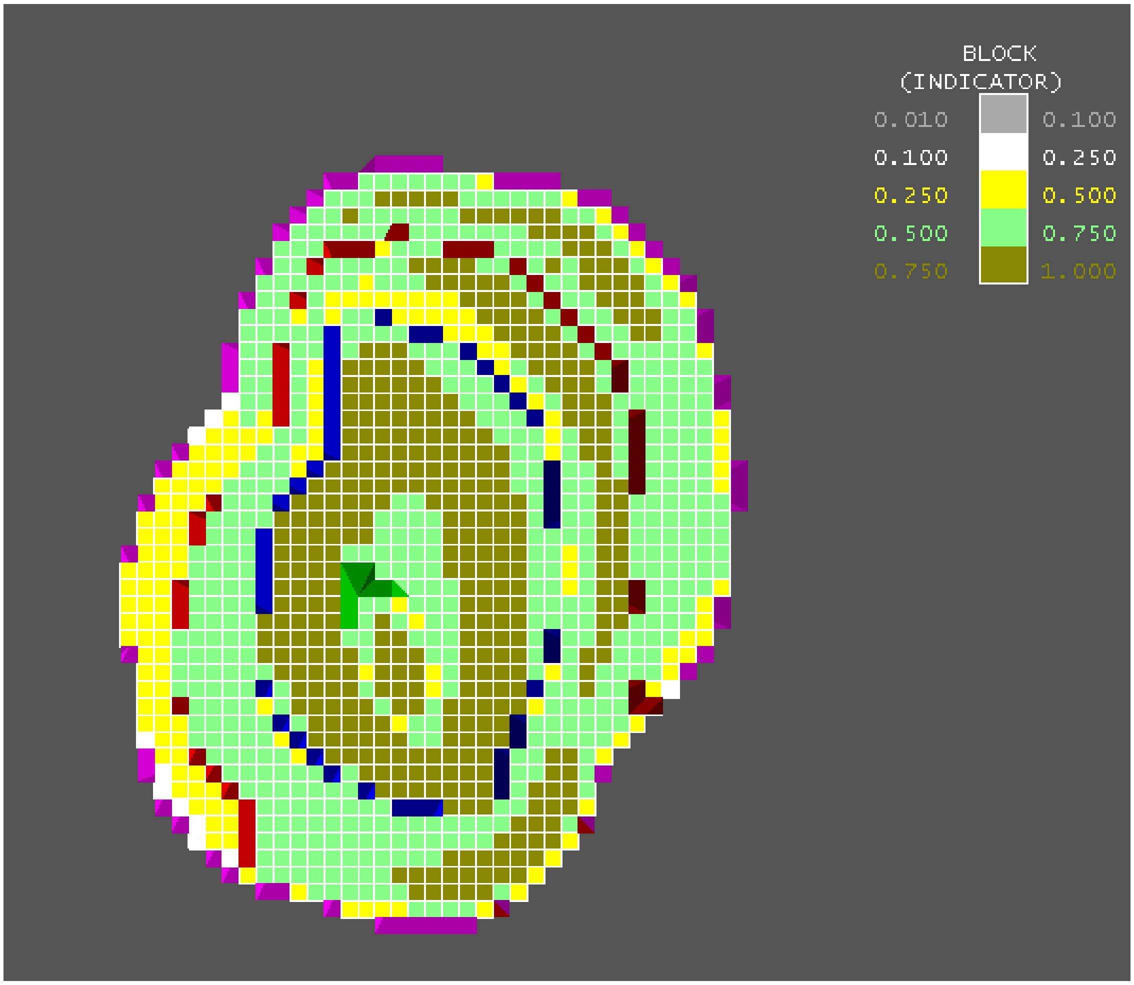


Figure 14 5410 bench mining probability map



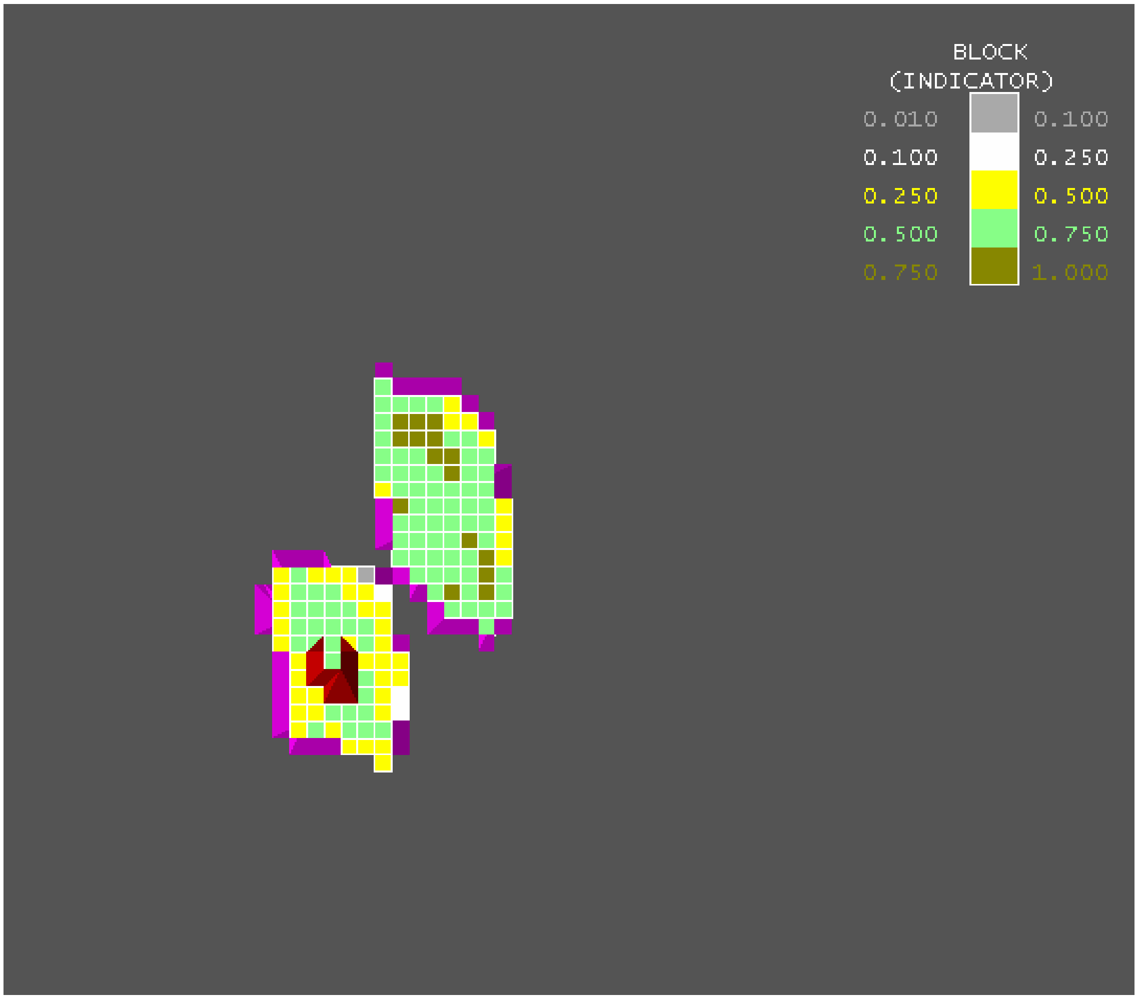
pulling in the NE walls of the phases.

- Reduce the size of the ultimate pit by moving in part of the west wall in the third and fourth phases.

Developing these conditional probability maps gives the mine planning engineer two distinct advantages over conventional planning.

1. Identification of zones sensitive to the natural variability of the mineralization aids in the positioning of intermediate phase and final walls.
2. This procedure followed here can also be used to target additional drilling when evaluating a deposit.

Figure 15 5010 bench mining probability map



Conclusions

It has been demonstrated that passing conditionally simulated grade models through the Four-X pit optimization software package gives the mine planning engineer an insight as to the likelihood of achieving the cash flow and production profile associated with a mine design based on an E-type or kriged estimate. It is recommended that the mining industry adopt such conditional simulation-pit optimization studies as a standard tool to analyze the inherent variability associated with grade on these aspects of design. If not, grade variability effects on mine design and cash flow like the reconciliation results represented in Figure 1 will continue.

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