Service variables in uranium disequilibrium studies

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Abstract — *The use of service variables in geostatistics (grade-thickness) is common in industry when developing block grade models and geologic resource estimates. This is particularly true in the case of sedimentary and stratabound-type deposits.*

The approach has proven merits when assessing ore reserves. However, there are cases where its application for preliminary data analysis produces biased estimates. One such case is encountered in uranium disequilibrium studies.

This paper demonstrates the inadequacy of service variables when performing uranium disequilibrium studies. Disequilibrium analysis is a necessary preprocessing step when radiation counts are used as raw data to calculate uranium reserves. Typically, a probe is used to obtain the gamma counts from open drill holes. These counts are then regressed (and corrected) against chemical assays obtained from core samples.

During this process the grade-thickness variable produces an optimistic image of the relationship between probe grades and chemical (true) grades, causing overestimation of the final geologic resource. The problem is analyzed through a case study.

Introduction

The use of service variables in geostatistics [grade-thickness (GT) and thickness (T)] is a common approach in industry when developing block grade models and resource estimates, particularly for sedimentary or stratabound-type deposits. Although this approach has proven useful when building grade models, it is sometimes misused when dealing with other problems related to ore reserve assessment.

In the case of disequilibrium studies of uranium minerals, there are usually two types of data available:

- Cored holes, from which chemical assays are obtained. This is regarded as the most accurate method for obtaining uranium grades.
- Open holes, where a radioactive count is obtained through a gamma probe.

The gamma counts, even after geophysical signal processing, are less accurate than the chemical assay. The readings are thus considered estimates of the actual uranium content, and they are susceptible to different types of biases.

One of the most significant sources of errors in gamma counting comes from disequilibrium, i.e., varying ratios of radioactive parent/daughter products in the ore. These ratios are, in turn, dependent on the reduction-oxidation state of the mineralization. In oxidized ore, the errors due to disequilibrium are more significant.

The procedure commonly applied to correct gamma logs is to regress on the variables defined for both types of grades, i.e., to obtain a general linear relationship for the whole deposit from a scatterplot of the gamma variable vs. the chemical variable. However, when a GT variable is used, and particularly after compositing, there is a significant amount of correlation introduced by the thickness component. Thus, the method gives a biased image of the relationship between the equivalent grades and the chemical assay grades. Further, and depending on the degree of disequilibrium, it is shown that the linear relationship between the grades themselves may be poor, so that a different approach may be needed to correct for disequilibrium.

The problem is analyzed using data from an arkose-hosted, stratified uranium deposit.

Uranium disequilibrium

The description of the geophysical processing of the gamma logs is beyond the scope of this work. It is assumed that the 'equivalent uranium grades' have already been obtained. This implies that signal attenuation factors, correction for dead time, calibration factors, and deconvolution algorithms (unsmoothing of the signal) have already been applied. A reference on the subject can be found in Scott, 1962, 1963.

The typical uranium decay chain includes several elements:

$$U_{238} \Rightarrow Th_{234} \Rightarrow Ra_{226} \Rightarrow Bi_{224} \qquad (1)$$

The radioactive minerals containing elements other than uranium are generally referred to as the 'daughter products' of the chain.

The gamma logger uses the Bi_{224} product to estimate uranium (Scott, 1962). The disequilibrium of U_{238} with respect to daughter products is mostly influenced by the reduction-oxidation state of the ore. U_{238} is likely to be leached out of the oxidized ore, hence resulting in a relative excess of daughter products. Thus, the gamma count tends to overestimate the actual uranium grade. The opposite is true for reduced ore, since the uranium leached from the oxidized zones is likely to be redeposited in a reduced environment. Further, if there are zones with mixed redox states, i.e., partly reduced and partly oxidized (in varying degrees), the ratio of parent-to-daughter products is erratic.

To correct for this disequilibrium, the redox state and the degree of oxidation are required. In many cases, it is feasible to predict the redox state of the orebody (oxidized or reduced), although this prediction will have a large error variance associ-

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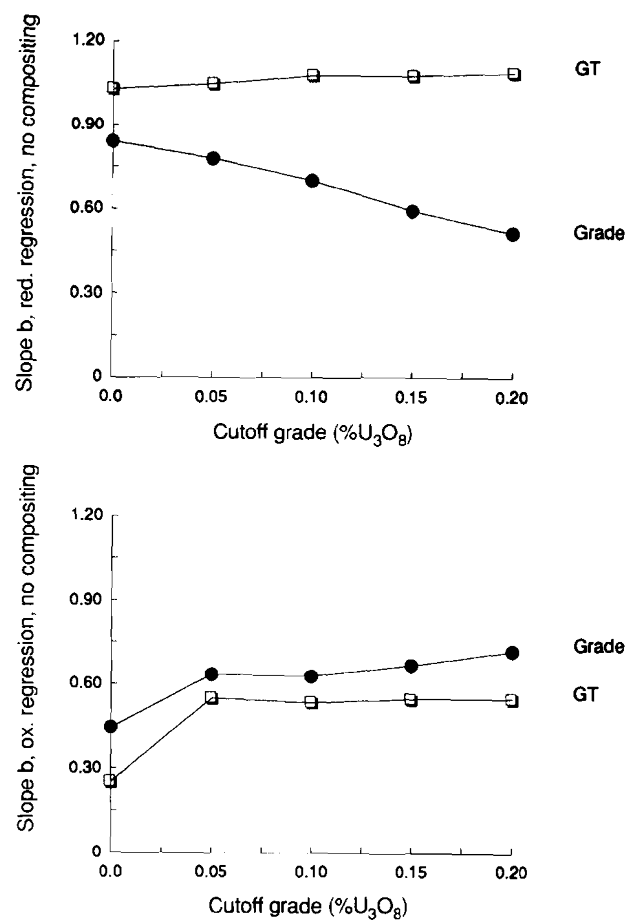

Fig. 1 — Slope b of the regression equation between equivalent data and chemical data for grades and GT. No compositing.

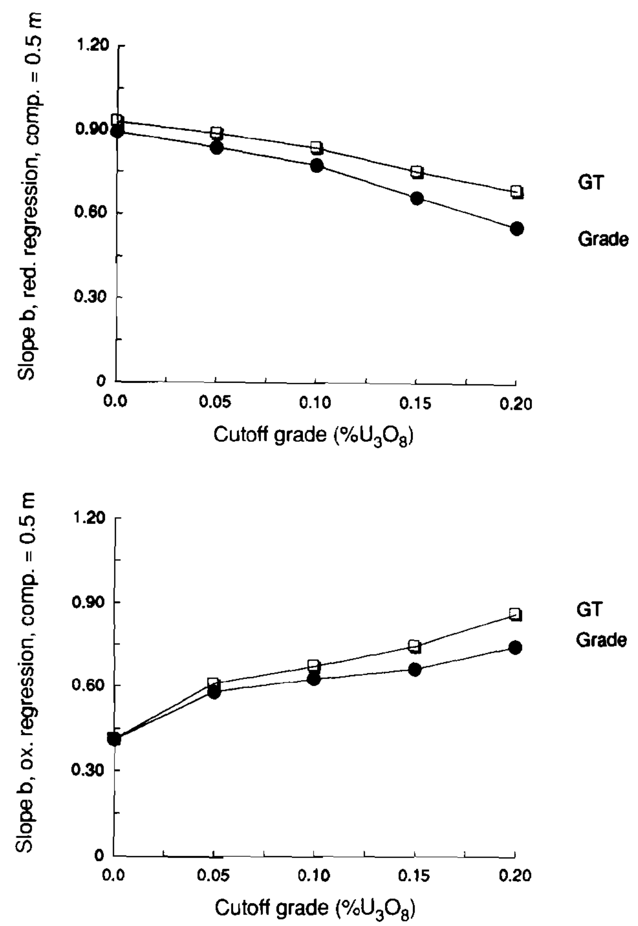

Fig. 2 — Slope b of the regression equation between equivalent data and chemical data for grades and GT. Composite length 0.5 m.

ated with it. Predicting the degree of oxidation is much more difficult.

Therefore, great care must be taken in using gamma log readings. First, the existence of disequilibrium must be established, and second, the ratio of parent-to-daughter products must be determined. If different redox states have been detected in the orebody, and the degree of oxidation varies from one area of the deposit to another, then the grades derived from the probe should be used only in the assessment of in situ resources, at a global scale. It would be inadvisable to use them if good accuracy at a local scale is required, such as in grade control at the time of mining.

Correlation and regression analysis

The equivalent grades (after preprocessing) are usually regressed using a linear relation derived from holes for which both cored data and gamma log readings are available. This is done by defining first the GT variable on both the chemical and the log data, obtaining the linear equation, and then retrieving the regressed grades. However, this approach is not supported by theory. It is not evident that the thickness variable is related to the ratio parent/daughter products, critical in the disequilibrium study.

To illustrate, assume that it is decided to correct gamma log readings through linear regression. Then, define the following variables:

chemG = the chemical grade (assayed), in percent
eqG = the equivalent (deconvolved) reading, in percent
chemGT = 'chemical grade times thickness'
eqGT = 'equivalent grade times thickness'

From classical statistics (Lapin, 1983, for example), the correlation between the chemical and equivalent variables (the equivalent being the predictor) is written:

$$\rho_{eqGT,chemGT} = \frac{Cov(eqGT, chemGT)}{\sigma_{eqGT} \bullet \sigma_{chemGT}} \qquad (2)$$

Also,

$$\rho_{eqG,chemG} = \frac{Cov(eqG, chemG)}{\sigma_{eqG} \bullet \sigma_{chemG}} \qquad (3)$$

In these equations, Cov(X,Y) is the covariance between the equivalent and the chemical variables, and σ represents the standard deviations of each variable.

Note also that:

$$eqGT = eqG \bullet T \text{ and} \qquad (4)$$

$$chemGT = chemG \bullet T' \qquad (5)$$

where the intervals (thicknesses) T and T′ are not necessarily the same.

The covariance in Eqs. (1) and (2) is defined as:

$$Cov\,(X,Y) = E\,\{X \bullet Y\} - E(X) \bullet E(Y) \qquad (6)$$

where E denotes 'expected value.' If X and Y are not correlated,

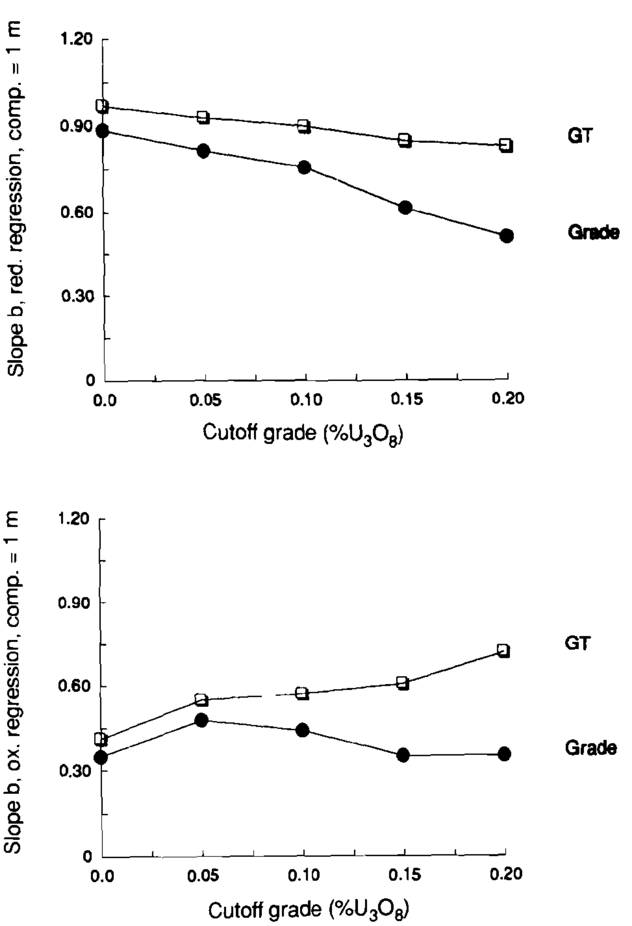

Fig. 3 — Slope b of the regression equation between equivalent data and chemical data for grades and GT. Composite length 1.0 m.

then E{X Y} = E(X)·E(Y), and the covariance is zero. The more related the variables X and Y are, the higher the covariance value and the correlation coefficient, according to Eqs. (1) and (2).

A similar development can be done in terms of the linear regression equation. If

$$\hat{Y} = a + b \cdot \hat{X} \tag{7}$$

represents the linear relation between X and Y, then the closer b is to 1 and a is to 0, the more similar X and Y will be. The ideal case is, of course, the 45° line, where Y = X. Thus, b is a critical parameter in the linear regression analysis, and it is estimated (under the least-squares criterion) by:

$$b = \frac{n \cdot \Sigma(X \cdot Y) - \Sigma X \cdot \Sigma Y}{n \cdot \Sigma X^2 - (\Sigma X)^2} \tag{8}$$

The following example analyzes the regression slope b as a function of redox states, composite lengths and cutoff grades.

The case study

The example is drawn from a real-life case. The deposit is a stratabound-type orebody, with uranium mineralization hosted in an arkose-mudstone, sub-horizontal sequence. About 35% of the data available is oxidized ore, 50% is classified as reduced ore, and 15% is mixed, with various degrees of oxidation. For simplicity, the ore classified as mixed was not included in this exercise.

The orebody has been drilled in an almost regular grid, the tightest drill-hole spacing being 25 m. Some 45 of these exploration holes have both chemical assays (from cores) and gamma logs. These 45 holes have been used in the following study.

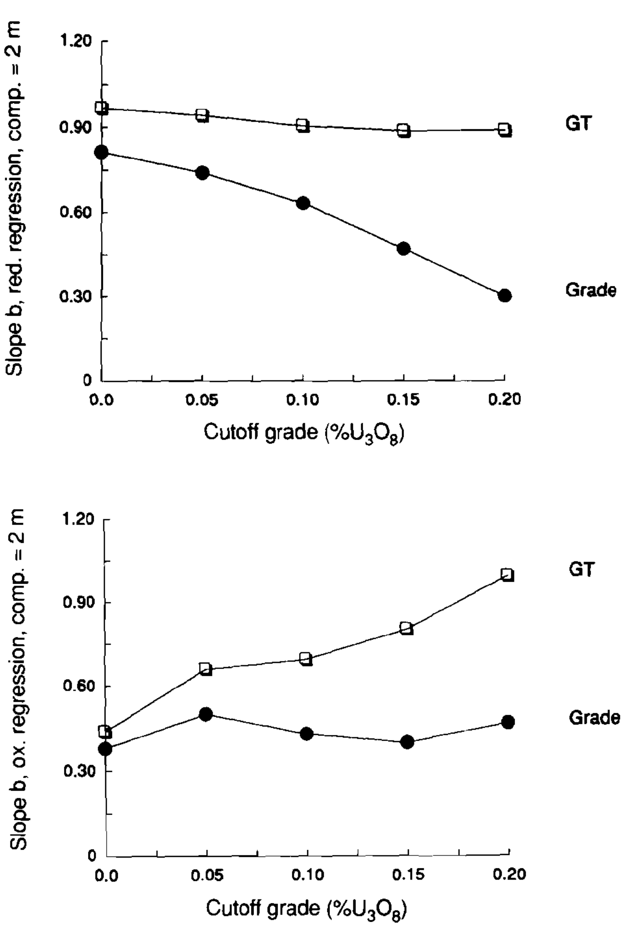

Fig. 4 — Slope b of the regression equation between equivalent data and chemical data for grades and GT. Composite length 2.0 m.

The first part of the exercise analyzes the relationship between equivalent grades and chemical grades, and equivalent GT vs. chemical GT, using the slope b [Eqs. (7) and (8)]. This is done for different grade cutoffs and different composite lengths, since it is important at the feasibility stage to understand how the in situ reserves are affected by economical and technical factors (dilation, cutoff and mining selectivity).

The study was performed by separating the chemical assay data from the gamma log data, and by also separating the intervals classified as oxidized ore from those classified as reduced ore.

Regression analysis

A regression equation of 'equivalent' data vs. chemical data (using the equivalent data as the predictor) was found for GTs and grades, for each type of ore (reduced or oxidized), for four different composite lengths (no compositing, 0.5 m, 1 m and 2 m), and for five different cutoff grades (no cutoff, 0.05%, 0.10%, 0.15% and 0.20% U_3O_8). The 'no compositing' (where the assay interval was multiplied directly by its corresponding grade to obtain the GT variable) and the 0.0% cutoff cases are used as base reference cases. Thus, a total of 80 linear regression equations was developed.

Figures 1 through 4 show the relation between the slope b of the regression vs. cutoff grade for both GT and grades, and for

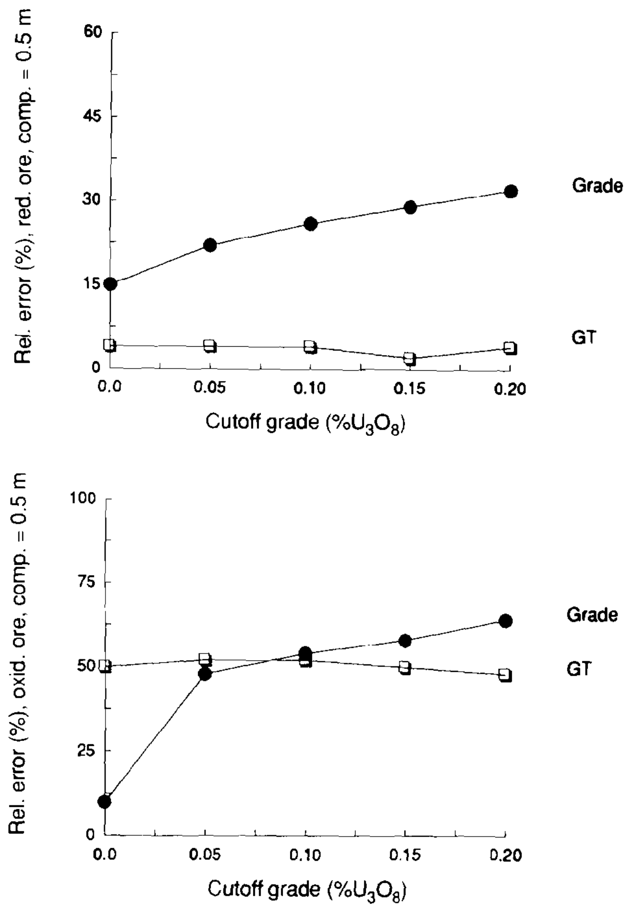

Fig. 5 — Relative error (%) of average grades. Composite length 0.5 m.

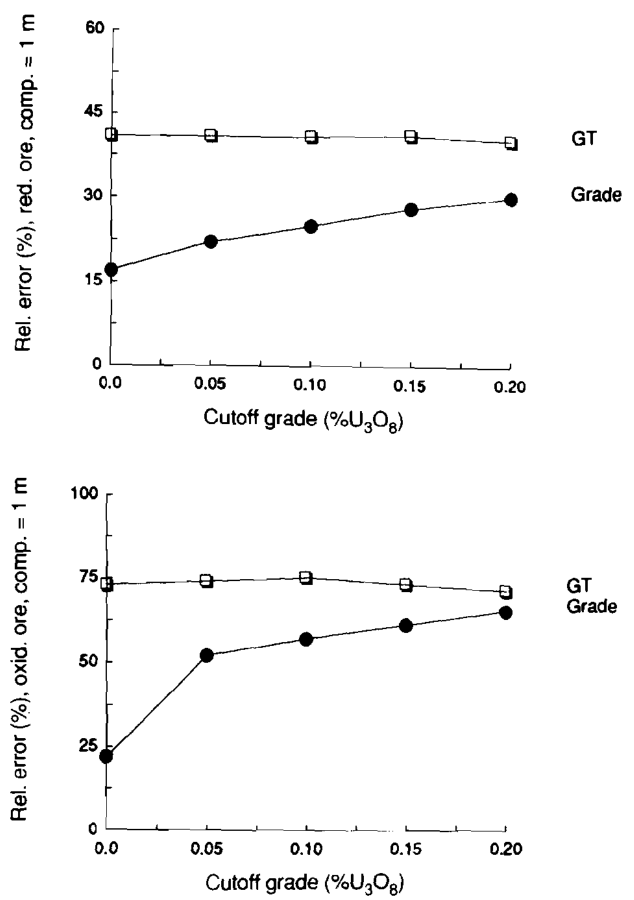

Fig. 6 — Relative error (%) of average grades. Composite length 1.0 m.

oxidized and reduced ore.

Figure 1 shows the 'no compositing' case, i.e., the thickness applied to the grade is the interval defined down the hole. In the case of the chemical assays, it is highly variable, although many are about 0.5 m long. In the case of gamma logs, most of them are 20 cm long, the interval length being defined during the signal processing stage.

It is seen in Fig. 1 that, for reduced ore, the slope of grade regression deteriorates rapidly with increasing cutoff, while the slope of the GT regression remains virtually unchanged and close to 1. In the case of oxidized ore (bottom of Fig. 1), the slope is in all cases poor, with the grade regressions being slightly better.

Figure 2 (reduced ore, composite length 0.5 m) shows that the slope of both GT and grade regressions deteriorates with grade cutoff, grade slope being always smaller than GT slope. In the case of oxidized ore (Fig. 2), the slope improves faster with increasing cutoffs for GT than it does for grades. Similar conclusions can be made about Figs. 3 and 4.

Notice that the effect of increasing composite length is to 'stabilize' the slope values of GT regressions for increasing cutoffs (reduced ore), or to bring it closer to 1 (oxidized ore). This effect is not as noticeable in all grade regressions, where the value of b falls quickly with cutoff grade.

Another interesting point to be made is that the slope of GT in both the reduced and the oxidized ore increases with composite length. Note that in Fig. 2 (composite length 0.5 m), the GT slope drops from 0.9 to about 0.75. In Fig. 4 (composite length 2 m), the GT slope is almost constant at about 0.9. Further, note that the grade slope (refer to the same figures) decreases with increasing cutoff much faster at a 2-m composite. This is to be expected since the occurrence of long composites with high grade is more erratic.

From the previous figures, it is evident that the GT variable has a more stable behavior in terms of regression equations than the grade variable. The difference is attributed to the length (thickness) incorporated into the GT variable. The correlation between the composite lengths themselves in the grade-thickness variable becomes more important as the composite length is increased. Thus, most of the goodness of the correlation seen between the equivalent GT and the chemical GT is due to the thickness component, rather than the grade component. And in the case of a disequilibrium study, thickness is not relevant to the problem. The impact of this discrepancy is tested in the next section through a case study.

Cross validation

The cross validation exercise described here is based on the fact that, for a group of drill holes, the 'true' (chemical) grades and the equivalent (gamma log) grades are known at the same spatial location. Thus, it is possible to compare the regressed grades against the true ones and calculate the corresponding error.

To avoid ambiguities, the transitional ore was not used in the exercise. The reduced and the oxidized data were separated in two different data sets. The first data set was used to calculate the regression equations for both the oxidized samples and the reduced samples for the grade and the GT variables. Table 1 shows the regression equations applied in each case. Notice that

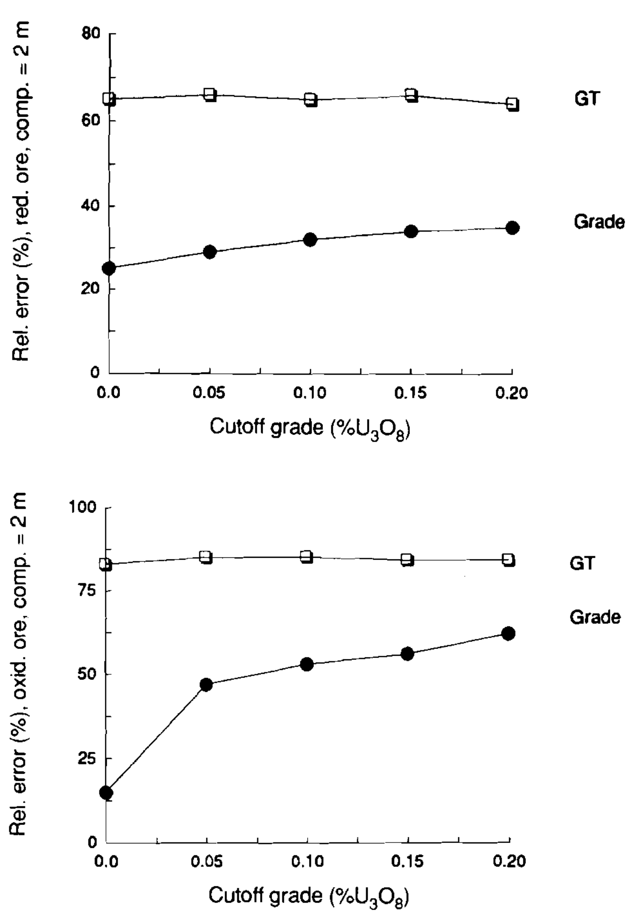

Fig. 7 — Relative error (%) of average grades. Composite length 2.0 m.

the regression parameters in the oxidized case are poor. This implies that the predicted grade will be poor and that a different approach may be more appropriate.

The equations of Table 1 were then used to correct the gamma counts in the second set of samples. Then, the corrected samples (for both the grade and the GT variable) were compared to the chemical grades at the same location. Thus, two sets of relative errors were obtained for each composite length, one for reduced ore, the other for oxidized ore (Figs. 5-7).

Table 1 — Regression slopes applied to the equivalent probe grades.

Ore type	Composite length	Regression equation
Oxidized	0.50	Y = 0.411X
Reduced	0.50	Y = 0.892X
Oxidized	1.0	Y = 0.349X
Reduced	1.0	Y = 0.884X
Oxidized	2.0	Y = 0.378X
Reduced	2.0	Y = 0.816X

Conclusion

The regression method, as applied here to gamma log data, leads to severe overestimation of the average in situ grades. This bias is generally greater when applying the regression equation to the grade-thickness service variable, an approach frequently used in industry.

Compositing to fixed interval lengths before correcting for disequilibrium adds the thickness correlation to the GT variable, which may yield a misleading (optimistic) image of the relationship between equivalent proven data and chemical data. The GT results are virtually unaffected by applying a cutoff, and they are generally worse than the grade results. Figures 5, 6 and 7 show overestimation levels that are unacceptable.

Another interesting observation is that even if the grade regression produces fairly good results when no cutoff is applied, truncating the distribution has the effect of upsetting the balance between the number of overestimations and underestimations. Thus the relative error increases rapidly. This is most notable for the oxidized ore.

The use of service variables (grade-thickness) should be avoided if a clear understanding of the relationship between probe and chemical grades is sought. Further, the use of traditional regression methods may lead to unacceptable overestimation errors, particularly after a cutoff is applied. ◆

Acknowledgments

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