Conditional simulations applied to recoverable reserves

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Abstract

A very common and long-standing problem in operating mines is the achievement of predicted grades and tonnages at the mill head as recovered from the mining operation. The discrepancies found between what is expected according to the Long-term block model and mine production are frequently significant. For evaluation and planning purposes, it is desirable to minimize these discrepancies. There are several factors affecting these differences, which include the smoothing effect of a moving average estimator such as kriging, the SMU being variable in volume from blast to blast in the sequence mined, sometimes varying due to geological factors, sometimes due to the geometry of the pit, or sometimes due to the local distribution of the mineralization, and other aspects of the volume-variance problem. Thus, typically the predicted grade-tonnage curves do not match the grades and tonnages of the material recovered.

Conditional simulations can be used to assess the uncertainty of the predicted grade in terms of minable reserves. For Open Pit mines, this is done by interpreting the optimization process as a Transfer Function between the Long-term block model and the material predicted to be selected within the pit. The usefulness of using any popular pit optimizer in conjunction with several simulated grade models of the deposit, yielding corresponding possible minable reserves, is discussed. The more refined assessment of uncertainty and sensitivity analyses using conditional simulations performed for deposit evaluation or development planning comes only with at a higher labor and time cost. This paper will discuss the practical situations where the extra effort is deemed to pay for the extra costs, as well as practical issues regarding the successful implementation of the procedures involved. The work presented here has been developed for open pit mines, but the concepts are also applicable to underground operations.

Introduction

In the mining industry block models are typically developed as tools used to estimate the overall resources of a deposit, and to allow further processing for planning the extraction and mine schedule for the life of the mine. Most of the mining projects and/or operations will have clear separation between the geologists usually charged with building the geologic model and the block model, and the mining engineers, typically in charge of developing the mine plan. And frequently, there is insufficient communication between these two groups. What results is that there is little understanding of the uncertainties embedded in the Long-term model; mining engineers generally try to compensate for these uncertainties by doing sensitivity analyses at pit optimization phase, implemented in terms of the input parameters to the optimization process. For example, sensitivities are run looking at the effect of different slope angles on the overall pit shape and recoverable reserves, also on an overall increase or decrease in grade, in metallurgical recoveries, etc. Generally, these studies are arbitrary, particularly with respect to grade, since there is no link between these sensitivity analyses and the actual geology or grade(s) distribution within the deposit.

There are several popular optimization algorithms, which include the Lerchs-Grossmann 3D Graph Theory algorithm[1], implemented in Whittle 4D as the Nested Lerchs-Grossmann algorithm[2], several versions of the "cone" optimizer, etc. Regardless of what tool is used to develop a pit, further refinements can be made at a local scale to account for certain practical mining considerations. The final result is a mine plan that programs the extraction process, using an "optimized" scenario, describing how the orebody would be best mined in an economic sense and with proper consideration to the technical aspects of the extraction.

It has been shown that the block model estimated from exploration or development drill holes, if compared to mine production (based on blast holes), frequently shows significant discrepancies.[3] Further, if reconciliation is done against "Mill Feed" production figures, the discrepancies are even larger. This results in deviations from the planned economic "optimum", in some cases large enough to cause the operation to become subeconomic. For evaluation and planning purposes, it is desirable to understand the source of these errors and to minimize these discrepancies.

It is generally accepted that the main reasons for the poor reconciliations sometimes observed are related mostly to the uncertainties in the geologic interpretation and in grade estimation, with all its different facets. Many are the factors that can affect uncertainty in the estimation process, such as the change of support effect, insufficient drilling, bad sampling or preparation techniques, interpolation errors in building the block model, the smoothing effect of the moving-average interpolators in vogue, etc.[4] However, and regardless of the reasons, it is important to recognize that there are no "perfect" block models, in the sense that the grade-tonnage curve predicted has always a measure of error (uncertainty).

The subject of recoverable reserves and modeling uncertainty has been studied within the geostatistical community for many years. The most difficult aspect of the problem has been to assess the uncertainty of the block model at a local scale, particularly on a block-by-block basis, or tonnages corresponding a small production periods (monthly production, for example). Propositions such as using the kriging variance to obtain confidence intervals for individual blocks have failed miserably[5,6], with only a global assessment of uncertainty being possible, where the overall deposit grade and tonnages above cutoffs are given a confidence level.[7]

In addition, it should be noted that even if we had a "reliable" estimation variance or other means to assess uncertainty, including the complete "real" cumulative distribution for each block, this would not provide us with any measure of uncertainty about the *minable reserves*, obtained after pit optimization. According to theory, the optimization process would have to be linear if *in-situ* confidence intervals, obtained during the block model construction, are to be used as confidence intervals for minable reserves.[6]

In the last 10 years or so significant developments have been made in the area of geostatistical conditional simulations.[6,8,9] Theoretical developments, in addition to the availability of cheaper and more powerful hardware and software[10], have allowed practitioners to begin implementing large scale orebody simulations. These simulations have been used to date (with few exceptions) as either grade control tools, or to develop a "ground truth" image of the mineralization at a small scale[4,11]. Two case studies are described here, where simulations are used to assess block-by-block uncertainty, and a pit optimizer (Whittle 4D) is used to translate that uncertainty into minable reserves. This allows for a better sensitivity analysis of the economic and technical aspects of the operation.

Conditional simulations and models of uncertainty

The purpose of conditional simulations is to build alternative grade models with such characteristics that they can be used to assess the uncertainty in grade estimation on a block-by-block basis. Among those characteristics, they are required to honor the full histogram and variograms of the conditioning data (drill holes or blast holes), therefore honoring the spatial variability of the deposit as represented by the samples available. Blast hole data should be used for conditioning whenever possible, although it is possible to build such a model from exploration data alone.

Geologic features are usually built in directly into the simulation model, by utilizing geologic zones or domains. In a more sophisticated but complicated scenario, and depending on the simulation algorithm used, the geologic model can be simulated itself. All conditional simulations are built on very fine grids, as fine as possible given practical time constraints and the hardware available. By honoring the histogram, the model correctly represents the proportion of high and low values, the mean, the variance, and other statistical characteristics of the data. By honoring the variogram, it correctly portrays the spatial complexity of the orebody, and the connectivity of low and high grade zones. These are fundamental pieces of information at the mine design, mine planning, and scheduling phases of the project.

When a number (for example 10) of these conditional simulations have been run and checked, then, for each block defined in the grid, there are the same number of possible block models available (10). These set of grades for each block, all equiprobable by construction, amount to have 10 different block models, and are interpreted to describe the model of uncertainty for each block, generally arranged as a *posterior cumulative conditional probability* (ccdf) curve. Preferably, a large number of simulations are needed to describe this curve better; however, due to practical limitations, a much smaller number is generally used as an initial approximation.

A reasonable grid for the simulation may be anywhere between 1m by 1m (for smaller and more erratic distributions) to 5m by 5m (for larger, bulk-tonnage or porphyry deposits). Such a high resolution is possible because of the random aspects of the algorithms used (conditional simulations are Monte-Carlo-based techniques). The data used to condition the simulations have to belong to the same population. Shift in the attitude of the ore controlling structures requires the separation of the data in different populations, as would geologic or lithologic boundaries. As mentioned above, often the same geologic boundaries used to constraint the grade estimation for a block model is used to constrain the conditional simulations. Thorough knowledge of the behavior of the high-grade population is required to control high grades in the simulation.[12] Issues such as limiting the maximum simulated grade should be carefully considered, since they have a significant impact on the uncertainty model the results.

The simulation algorithm should be decided based on the type of the deposit, the available data set and the desired output. The first decision is whether to use a parametric or non-parametric approach. Examples of each are the Sequential Gaussian[9], and Sequential Indicator[8] simulations. The latter is more complicated, based on multiple indicator kriging techniques[6], and requires definition of several indicator cutoffs. The former is simpler and quicker, although more restrictive in its basic assumptions. Any available geological criteria (also known as soft information[13,10]) can and should be used. As with any estimation exercise, variograms have to be estimated and modeled, and a number of important parameters need to be considered. These include: minimum and maximum data value and simulated value allowed, number of conditioning data to be used, search distances, anisotropies, etc. All these parameters, along with the chosen algorithm and simulated domain have a bearing on the output simulations, and the range of possible values that any given block may have (uncertainty model).

Finally, it is very important to thoroughly check the simulated values. The histogram of the simulated data should be compared to that of the original conditioning data. Both should be similar in terms of simple statistics and overall shape of the histogram. The variograms of the simulated data should be similar to the input models, at least up to the search distance used. The original conditioning data and the simulated data should be plotted on maps at the same scale. Close examination of the two sets of maps will detect any possible deviations of the simulated data from the conditioning data. It is also instructive to observe what the conditional simulation produces in areas where there is little or no original (conditioning) data.

The Transfer Function

In this paper the overall process of estimating a block model, and posterior mine planning and scheduling is seen as an integrated process. Consequently, pit optimization and posterior mine planning and scheduling work can be seen as a "transfer function"[6], representing key aspects of the mine feasibility study, or further mine development after the open pit is in operation. Since it is recognized that the block models always carry uncertainty, a "perfect" input of block grades into the pit optimizer is not possible. Hence, the idea is to input each of the simulations obtained above (or a subset of them, selected according to given criteria relevant to the problem under investigation) to obtain a "response distribution", i.e., a series of "optimal" pits, all equiprobable, that can be used to assess the impact of possible variations in grade at a local scale. Figure 1 illustrates the concept of a Transfer Function.

There are several instances where this concept may be applied. For example, frequently there is a need to assess to uncertainty of the block grades near the bottom of a proposed pit. Often the pit optimizer is driven downwards by a few high grade composites (samples), responsible for producing an area with high grade block estimates, which in turn pay off extended push-backs and digging to deeper levels of the deposit. A good understanding of the risks involved in taking that mineralization at face value is necessary to avoid surprises, particularly if the deposit is marginal, and the issue has to be resolved before mine development takes place.

Another example could include a case where an expansion is planned, in which case the extended mine plan calls for extraction of ore in areas where there may be less drilling and less geologic knowledge. The projected mine schedule should then be checked against a set of conditional simulations to see whether on a month-by-month basis the mine plan is feasible. Yet other examples may include the mine plan calling for extended periods of mostly waste mining (push-backs). The plant or heaps are planned to be fed with ore previously mined and stockpiled. In such case, a tight mine scenario could develop, and it would be important to know how much risk is involved if either the grades stockpiled are not the planned grades, or if the grades found after completing the push-back are not as planned, or if there could be more waste material than planned that has to be removed. This problem also calls for a detailed (and local) assessment of grade uncertainty, and possibly a modified mine plan that will have some flexibility embedded into it.

Therefore, despite the increased labor and effort of this procedure, it appears that the availability of alternative simulated block models (equiprobable images of grade distribution) can have significant impact in improving pit optimization, mine plans and schedules, and in the overall analysis of the mine economics.

Case study No. 1

This case study was presented at the 1997 Optimizing with Whittle Conference, help in Perth, W. Australia, April 8-9.[14] It is summarized here as an example of one application of conditional simulations for operating mines.

The work was done for a well-known bulk-tonnage Au deposit in central Nevada, that has been in operations for a number of years. There are three processing units (possible destinations for ore out of the pit), dependent mostly on ore grade, and partially on oxidation state; the first is mill feed; the second is re-usable, crushed heap leach pads; and the third is run-of-mine dump leach.

The purpose of the work was to verify the grade-tonnage curves predicted by the block model, in this case by re-running optimum pits for each of the 5 conditional simulations run. In addition, the pit optimization procedure included a sensitivity analysis on metal content for each block (-10% and +10% of overall metal content), for comparison with the simulations.

The main conclusion reached is that the overall shape of the pit does not change for any of the conditional simulations (the deposit is well drilled and well known), but the resulting grade-tonnage curves do. Table 1 shows the results for the kriged block model (currently used for Long-term planning), the 90% and 110% metal sensitivities, each of the 5 simulations, and the minimum, average, and maximum simulation for each of the destinations (dump leach, crushed leach, and mill feed). It is clearly seen that the block model, according to the simulation model and due mostly to smoothing, significantly underestimates mill feed tonnage (somewhat underestimating grade), significantly overestimates tonnage and grade of crushed heap leach ore, underestimating again the lower end of the grade-tonnage curve, the run-of-mine dump leach ore.

The results in Table 1 are presented in the form of factors to protect confidentiality, and are a useful tool to aid in Long-term mine planning, when planning mill feed and pad size and usage, and scheduling ore through the processing facilities.

Table 1: Summary of Optimum Pit Scenarios by Process

Process	Kriged Block Model	90% Metal	110% Metal	Simul. Minimum	Simul. Average	Simul. Max.
ROM Tons	100%	94%	111%	110%	125%	131%
ROM Grade (oz)	100%	100%	97%	92%	93%	93%

ROM Ounces	100%	95%	107%	89%	109%	115%
Leach Tons	100%	97%	102%	80%	85%	93%
Leach Grade (oz)	100%	100%	97%	93%	94%	96%
Leach Ounces	100%	97%	102%	76%	82%	89%
Mill Tons	100%	94%	119%	108%	116%	120%
Mill Grade (oz)	100%	100%	100%	98%	101%	108%
Mill Ounces	100%	94%	120%	107%	119%	130%
Total Ounces Recovered	100%	96%	108%	87%	98%	106%
NPV @ 10%	100%	101%	98%	87%	96%	101%

Case study No. 2

The second case study shows a different application of conditional simulations. The work was performed on a small Au orebody, called the Toyita-Norita prospect, associated to the same structural trend of Niugini Mining's known San Cristóbal mine, in Region II, near Antofagasta, northern Chile.

The mineralization is structurally controlled, and the small orebody is approximately 600m long and 150m wide, known to a depth of about 200m. It is hosted within a system of mineralized structures, with a small halo of kaolinitic alteration, and to a minor extent, sericitic. Host lithologies include diorites, graindiorites, monzodiorites, and fine-grained granites. In addition, a breccia associated with the main structure is present, and a favorable host for mineralization. Gold occurs in quartz-pyrite veins, generally striking N30W, which presents also abundant jarosite, goethite, hematite and manganese oxide.

The database available was composited to 5m bench composites, and is based on the assumption that the orebody will be mined using the same methods and equipment as the nearby San Cristóbal mine. Figure 2 shows the histogram of the 5m composites, with a coefficient of variation of 5.26, and a maximum grade of 73 g/t.

In addition to the histogram, indicator variograms were run on 7 indicators, in order to prepare for an indicator-based simulation algorithm. These variograms represent the spatial correlation of the Au mineralization at different grade cutoffs, including mineralization cutoff (0.2 g/t), a potentially economic cutoff (0.45 g/t), and higher cutoffs.

The geologic modeling of Toyita-Norita is based on a simple grade envelope (at 0.2 g/t), used to delineate the mineralized area and to constrain interpolation. The geologic envelope defined a total of 9,913 blocks of size 5x5x5m, used to create the block model. The simulations described below were also constrained by this grade envelope.

In addition to the creation of the block model for later pit optimization and mine planning, a set of 10 conditional simulations were run on Toyita-Norita. The objectives of these simulations was to assess the risk involved in the block model-defined resources and reserves, i.e., the grade-tonnage curve. In addition, of particular concern was the fact that most of the high grade ore was present at the bottom of the pit, and was interpolated from a few high grade composites, clustered in the center-bottom of the deposit. The risk analysis performed is based on the idea that grade variations within the high grade zone could affect both the overall grade-tonnage curve, and the position of the final optimized pit.

The simulation algorithm used in this case was the Sequential Indicators Simulation, which has been successfully applied for grade control at the San Cristóbal mine itself. As stated above, reproduction of the original composite statistics (histogram and indicator variograms in this case) is a requirement, and after running the 10 simulations, each one of them was checked for good reproduction of the overall statistics.

Each simulation thus yield a grade-tonnage curve that can be interpreted as alternative "block models", all equiprobable. These simulations represent a conditional cumulative distribution frequency curve (ccdf), which characterizes the overall uncertainty around the estimated resources/reserves. Table 2 shows the tons and grades predicted from each simulation, along side the in-situ geologic resources predicted by the block model, all at 0.60 g/t cutoff, which corresponds approximately to the economic cutoff for this property.

Table 2: In-situ Reserves, 10 Conditional Simulations

Simulation No.	Tons (x1000.00)	Grade (g/t)	Contained Au (oz)
1	1206.40	1.74	67316
2	1158.45	1.84	68617
3	1188.60	1.77	67682
4	1155.17	1.77	65779
5	1255.46	1.82	73645
6	1251.47	1.71	68756
7	1258.00	1.76	71274
8	1198.78	1.61	62088
9	1143.55	2.09	76897
10	1166.80	2.06	77125
Average	1198.27	1.82	70016
MIK Estimate	1214.12	1.83	76476

Table 3 presents the summary statistics derived from Table 2. It can be seen that, on average, the block model and the averaged 10 simulations result in about the same grade, with slight differences in tonnage and contained gold. However, the variability from one simulation to the next can be large, both in terms of tonnage and grade. The variation in percent, given at the bottom of the Table, corresponds to the minimum and the maximum found for any simulation, regardless of each column.

Table 3: Comparative Statistics, arranged according to Contained Metal, In Situ Reserves

Statistic	Tons (x1000.00)	Grade (g/t)	Contained Au (oz)
Minimum (Contained Au)	1198.78	1.61	62088
20th percentile (Contained Au)	1155.17	1.77	65779
50th percentile (Contained Au)	1158.45	1.84	68617
80th percentile (Contained Au)	1255.46	1.82	73645
Maximum (Contained Au)	1166.80	2.06	77125
Simulation Average	1198.27	1.82	70016
MIK Estimate	1214.12	1.83	76476
Variation, %	+3.6; -1.3	+14.2; -12.1	+0.1; -18.2

In Table 4 the grades and tons *recovered* for each simulation (after pit optimization) are presented, for a cutoff grade of approximately 0.62 g/t that resulted from the assumed costs and recoveries, and ranked by contained ounces. This results from the pit optimization routine, and as such, they are termed here "minable" reserves.

Table 4: Minable Reserves, 10 Conditional Simulations

Simulation No.	Tons (x1000.00)	Grade (g/t)	Contained Au (oz)	Strip Ratio
1	577.96	2.29	42473	2.67
2	632.76	2.41	49029	2.67
3	678.14	2.40	52327	3.09
4	693.49	2.37	52843	2.76
5	696.28	2.39	53503	2.66
6	698.38	2.45	55012	2.87
7	730.14	2.39	56105	3.04
8	734.33	2.41	56899	2.92
9	831.51	2.16	57746	2.88
10	803.43	2.27	58637	2.85
Average	707.64	2.35	53547	2.84
MIK	707.54	2.36	53686	2.84

Table 5 summarizes the corresponding statistics, including variations (expressed as factors) for tons, grade, contained metal, and strip ratio. According to the simulation model, the possible range of contained ounces is about -20% and +10%, with respect to the block model estimate, which is very similar to the average of the ten simulations.

Note that the models of uncertainty summarized in Tables 3 and 5 are quite different. This is the effect of applying a transfer function (in this case, a pit optimization procedure); now it is evident that confidence intervals (or better yet, the full uncertainty model) that are applicable to in situ reserves cannot be translated directly into confidence intervals at the minable stage. This would only be true if the transfer function was linear. Obviously, the pit optimization and other mine planning practices are far from being linear, and so it is demonstrated in Tables 3 and 5.

Table 5: Comparative Statistics, Minable Reserves

Statistic	Tons (x1000.00)	Grade (g/t)	Contained Au (oz)	Strip Ratio
Minimum (Contained Au)	577.96	2.16	40137	2.66
20th percentile (Contained Au)	632.76	2.27	46181	2.67
50th percentile (Contained Au)	696.28	2.39	53503	2.85
80th percentile (Contained Au)	803.43	2.45	63287	3.04
Maximum (Contained Au)	831.51	2.45	65498	3.09
Simulation Average	707.64	2.35	53547	2.84
MIK Estimate	707.54	2.36	53686	2.84
Variation, %	+17.5; -18.3	+3.8; -8.5	+9.2; -20.9	+8.8; -6.4

Conclusions

It has been shown using two short examples that there is significant potential improvement in the understanding of difference between block model estimates and actual recoverable reserves. Conditional simulations allow to integrate mode Long-term estimation, Short-term models, pit optimization and mine planning.

In the case studies described, it was seen that the precision of the recoverable grade-tonnage curve from a Long-term model is difficult, and without conditional simulations there is little opportunity to study the sensitivity of factors such as the volume-variance relationship on the recoverable grade-tonnage curve. In addition, it has been shown that uncertainties related to in-situ reserves do not relate directly (linearly) to uncertainties in minable reserves.

In addition, conditional simulation is a tool that allows for a wide variety of optimization and engineering studies, not developed in this paper, that resolves the inherent shortcomings of other recoverable reserve estimation methods currently in vogue.

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